STAT_1302 ; Lecture 7 ; Jan. 30, '24

- Assignment 1 due tomorrow.
- Assignment 2 Coming Soon.

§ 9.4 Tests for the Population Proportion when the Sample Size is large

Set-up: A random Sample of Size n is drawn from a pop. Interest Centres on testing

$H_0: p = p_0$ vs. $H_1: p > p_0$ or $H_1: p < p_0$ or $H_1: p \neq p_0$ at level of Significance α.

Assumption: n is large. Verify these Conditions $n \times p_0 > 5$ and $n \times q_0 > 5$ where $q_0 = 1 - p_0$.

Test Statistic:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \qquad \text{Given}$$

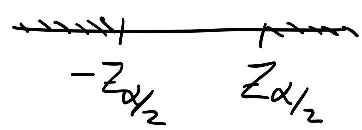
Assuming H_0 is true,

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \approx N(0,1).$$

(ie. this is a Z-test.)

Test	Rejection Region
$H_0: p = p_0$ vs. $H_1: p > p_0$	$Z > Z_\alpha$
$H_0: p = p_0$ vs. $H_1: p < p_0$	$Z < -Z_\alpha$
$H_0: p = p_0$ vs. $H_1: p \neq p_0$	$\lvert Z \rvert > Z_{\alpha/2}$

$\Leftrightarrow$



Ex. A five year old census recorded that 20% of families in a large community live below the poverty line. To determine if this percentage has changed, a random sample of 500 families is studied and 91 are found to be living the poverty line. Does this find indicate that the current percentage is different from the percentage obtained from the past

Census? Let $\alpha = 0.01$.

<u>Sol'n:</u>

1. Let p be the proportion of families living below the poverty line.

2. $H_0 : p = 0.20$

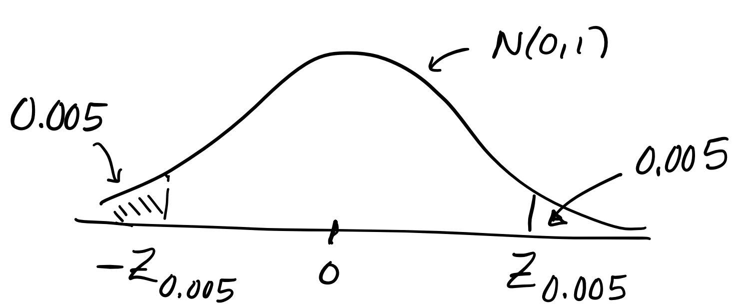
   $H_1 : p \neq 0.20$

3. $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.182 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{500}}} = -1.00$$

$$\hat{p} = \frac{91}{500} = 0.182.$$

3. Reject H_0 if $|Z| > Z_{\alpha/2}$.

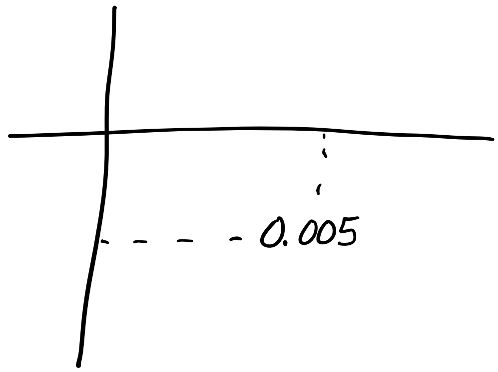
$\alpha = 0.01$; $\alpha/2 = 0.005$, $Z_{0.005}$



Look up 0.005 in Table IV in the area portion. This gives

$-Z_{0.005} = -2.58. \quad \therefore \quad Z_{0.005} = 2.58.$

Table IV



$\therefore$ Reject H_0 if $|Z| > 2.58.$

4. Decision: $|-1.00| = 1 < 2.58. \quad \therefore$ fail to reject H_0.

5. Conclusion: There is not enough evidence to suggest that the percentage of families living below the poverty line has changed since the last Census.

Assumption: n is large because

$n \times P_0 = 500 \times 0.20 = 100.0 > 5$ and

$n q_0 = 500 \times 0.80 = 400 > 5.$

We can also test $H_0: p = 0.2$ vs. $H_1: p \neq 0.2$ using a 99% CI for p.

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.182 \pm 2.58 \sqrt{\frac{0.182 \times (1 - 0.182)}{500}}$$

$$= (0.14, 0.23).$$

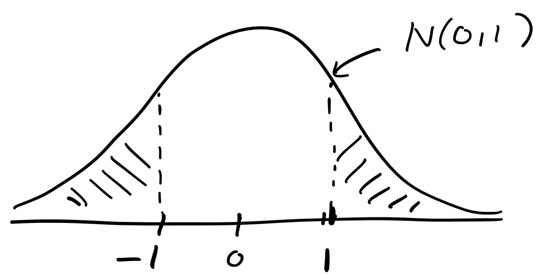
Decision: Since $p_0 = 0.2$ is in the CI for p, we fail to reject H_0. Same conclusion as before.

Using the p-value approach:

Test	P-value
$H_0: p = p_0$ vs. $H_1: p > p_0$	$P(Z > Z_{obs.})$
$H_0: p = p_0$ vs. $H_1: p < p_0$	$P(Z < Z_{obs.})$
$H_0: p = p_0$ vs. $H_1: p \neq p_0$	$P(\lvert Z \rvert > \lvert Z_{obs.} \rvert)$.

Back to our example.

$H_0: p = 0.2$ vs. $H_1: p \neq 0.2$. Let's test using the p_value approach.



[p-value is $P(Z < -1) + P(Z > 1)$.]

$$
\begin{aligned}
\text{P-value} &= 2 \times P(Z < -1) \\
&= 2 \times 0.1587 \quad \text{(from Table IV)}. \\
&= 0.3174 > \alpha = 0.01
\end{aligned}
$$

$\therefore$ fail to reject H_0; Same conclusion as before.

Recall: For any hypothesis test, reject H_0 if

$$\text{P-value} < \alpha.$$

Assumption: Same assumptions as critical value approach when testing using the p-value. Here, $n \times p_0 > 5$ and $n \times q_0 > 5$.

(Read ebook § 9.4 for additional two-sided alternative p-value calculations.)

Ex. Test $H_0: p = 0.44$ vs. $H_1: p < 0.44$ at $\alpha = 0.01$.
Given: A random sample of 450 observations and $\hat{p} = 0.39$. (State any assumptions you are making.)

Rejection Region:

1. Parameter: p, pop. proportion

2. $H_0: p = 0.44$ vs. $H_1: p < 0.44$

3. test stat.

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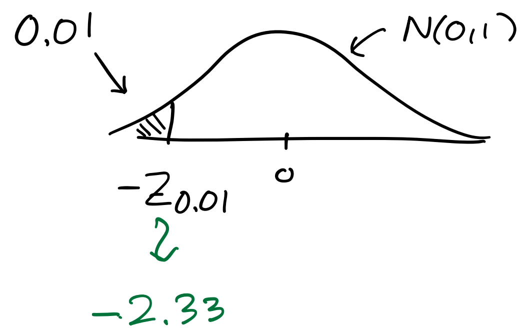
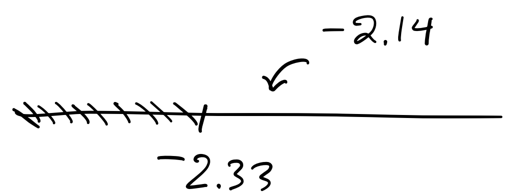
$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.39 - 0.44}{\sqrt{\frac{0.44 \times 0.56}{450}}} = -2.137 \approx -2.14$$

$$q_0 = 1 - p_0 = 1 - 0.44 = 0.56$$

4. Reject H_0 if $Z < -Z_{0.01}$.



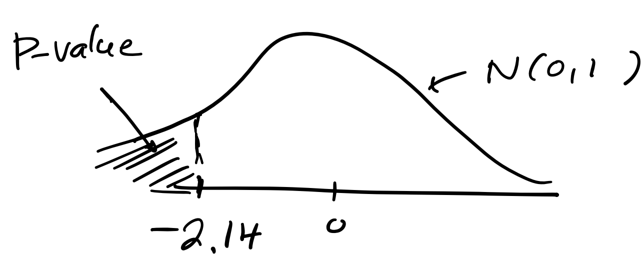
Reject H_0 if $Z < -2.33$.



5. Fail to reject H_0 because -2.14 is not in the rejection region.

6. Conclusion: Pop. prop. is 0.44.

P-value Approach to testing:



$$\text{P-value} = P(Z < -2.14) = 0.0162 \text{ (Table IV)}.$$

P-value $= 0.0162 > \alpha = 0.01$ $\therefore$ fail to reject H_0.

Same conclusion as before.

End of §9.4.