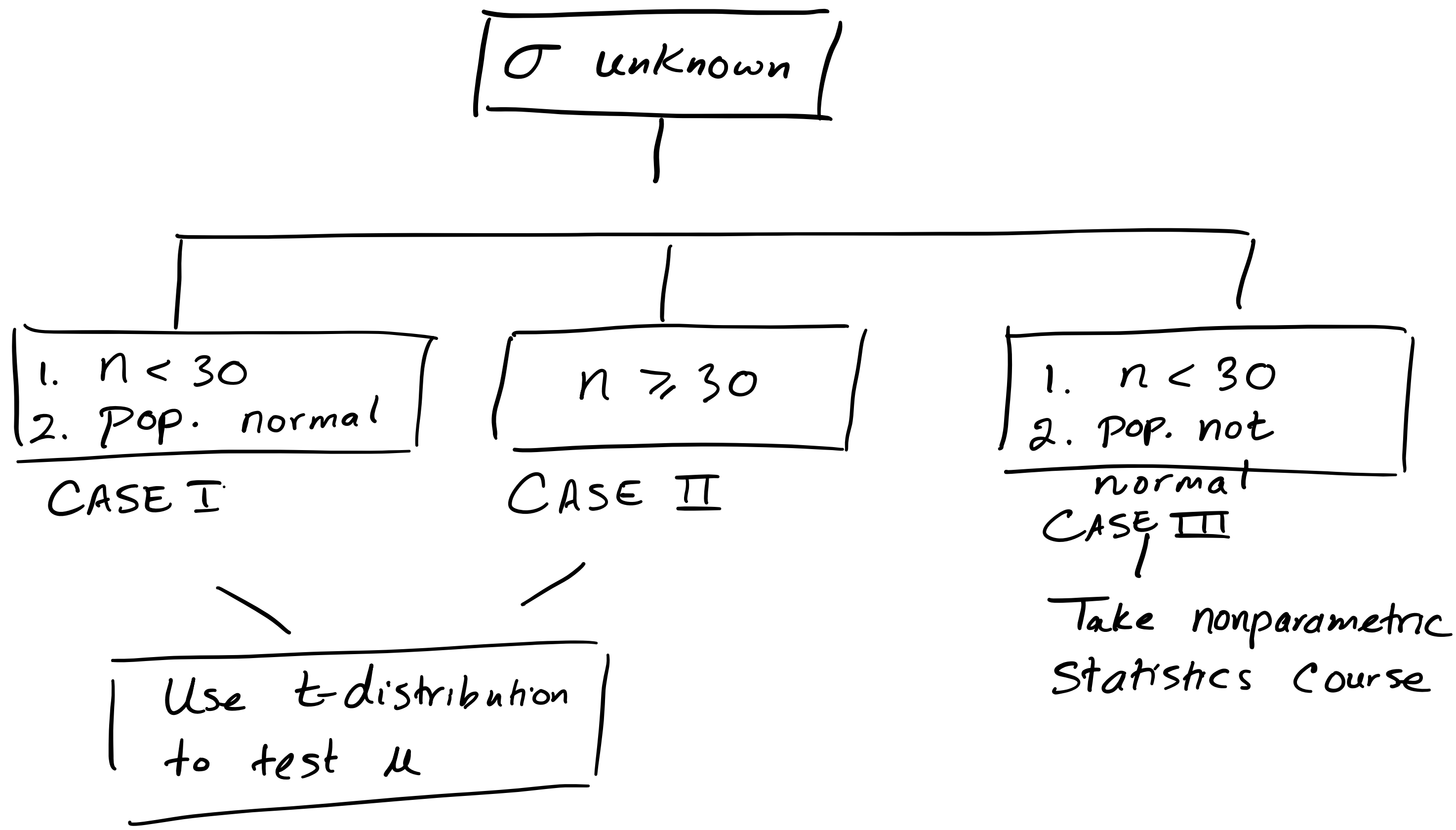
STAT 1302 ; Lecture 6 ; Jan. 25, '24

## §9.3 Testing for $\mu$: $\sigma$ is unknown




Set-up: In all three cases, we have a random sample, $X_1, \ldots, X_n$ from a population with mean $\mu$ and standard deviation, $\sigma$. $\sigma$ is unknown. $X_1, \ldots, X_n$ are independent (i.e. $\frac{n}{N} < 0.05$).

To test $H_0: \mu = \mu_0$ ($\sigma$ unknown) in Cases I & II, use the test statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \leftarrow \text{Given}$$

Under $H_0$ (ie. assuming $H_0$ is true),

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{n-1}.$$

Here,

$\bar{x}$ = Sample mean,

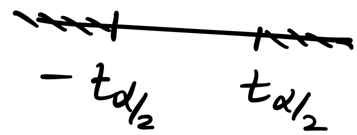
$\mu_0$ is the value $\mu$ under $H_0$,

$s$ is the Sample std. dev'n,

$n$ is the Sample size.

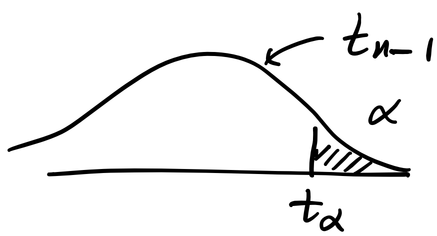
| Test ($\sigma$ is unknown) | Rejection Region |
| --- | --- |
| $H_0: \mu = \mu_0$, $H_1: \mu > \mu_0$ | $t > t_\alpha$ |
| $H_0: \mu = \mu_0$, $H_1: \mu < \mu_0$ | $t < -t_\alpha$ |

- $H_0: \mu = \mu_0$, $H_1: \mu \neq \mu_0$ $\qquad |t| > t_{\alpha/2}$




$df = n-1$.

$t_\alpha$ means $t_{n-1;\alpha}$




- Can use a $100(1-\alpha)\%$ CI for $\mu$ where $\sigma$ is unknown to test $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$ as follows.

Approach: Compute $\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

i) If $\mu_0$ is in the CI for $\mu$, fail to reject $H_0$.

ii) If $\mu_0$ is not in the CI for $\mu$, then reject $H_0$.

Omit p-value computations for the t-test.

Ex. The mean drying time of a brand of spray paint is known to be 90 seconds. The research division of the company believes that adding a new chemical ingredient to the paint will accelerate the drying process. To investigate, the paint with the chemical ingredient is sprayed on 15 surfaces. The mean and standard deviation from these drying times are found to be 86 and 4.5 seconds, respectively. Do these data provide strong evidence that the mean drying time is reduced by the addition of the new chemical? Let $\alpha = 0.05$. State any assumptions you are making.

Sol'n

1) $\mu$: mean drying time

2) $H_0: \mu = 90$

$H_1: \mu < 90$

$\sigma$ is unknown.

3) test stat.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{86 - 90}{4.5/\sqrt{15}} = -3.44$$

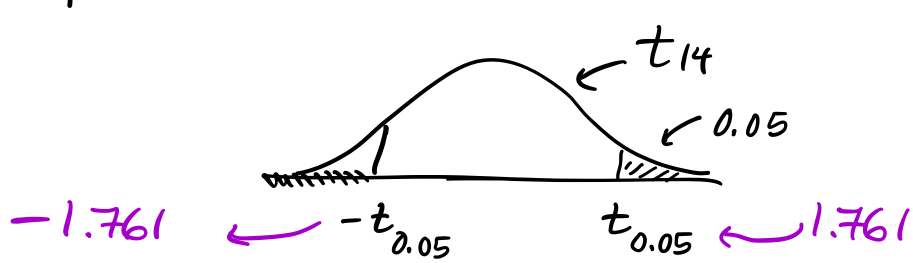
4) Rejection Region:

Reject $H_0$ if $t < -t_\alpha$ ; $df = n-1 = 15-1 = 14$.

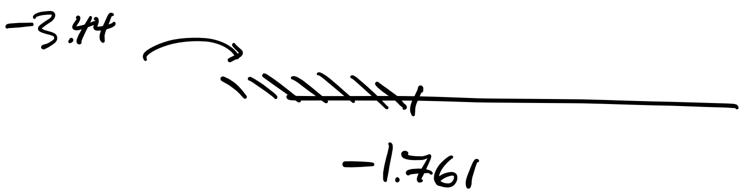
$\alpha = 0.05$

| df | 0.05 |
|---|---|
| ⋮ | ⋮ |
| 14 | 1.761 |

← Table V (App. B).



Reject $H_0$ if $t < -1.761$




5. Reject $H_0$ since $t = -3.44 < -t_\alpha = -1.761$.

6. Conclusion: We are 95% confident that addition of the new chemical reduced the drying time.

→ Assumption: $n = 15 < 30$ so pop. normal; $\frac{n}{N} < 0.05$.

Jargon: In hypothesis testing, when we reject $H_0$ in favour of the $H_1$ at level $\alpha$, we say that the result is statistically significant.

Ex. A maker of diet meals claims that the average calorie count of its meals is 800. A researcher tests 12 meals and find the average number of calories to be 873 and the standard deviation to be 25. Does the evidence support the claim? Let $\alpha = 0.05$. State any assumptions you are making.

<u>Sol'n</u>:

1. Let $\mu$ be the mean calorie count

2. $H_0: \mu = 800$

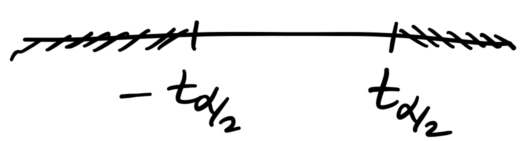
   $H_1: \mu \neq 800$

3. $\sigma$ unknown ; $S = 25$.

   test stat. : $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} = \dfrac{873 - 800}{25/\sqrt{12}} = 10.12$

4. Rejection Region:

   Reject $H_0$ at level $\alpha$ if $|t| > t_{\alpha/2}$




$d.f. = n - 1 = 11$

$\alpha = 0.05$

$t_{11;\,0.025} = 2.201$ from Table V (App. B).

Reject $H_0$ if $|t| > 2.201$

5. Decision: Since $|10.12| = 10.12 > 2.201$, we reject $H_0$.

6. Conclusion: We are 95% confident that the mean calorie count of the diet meals is not 800.

Assumption: Since $n = 12 < 30$, need normal popul'n. And $\frac{n}{N} < 0.05$.

Ex. E-Canis infection is a tick-borne disease of dogs that is sometimes contracted by humans. In the general population, the mean white blood cell count is $7250/mm^3$. The

The mean white blood cell count of a

random sample of 15 infected persons is $4767/mm^3$ and the standard deviation is $3204/mm^3$. Does the data suggest that persons infected with E. Canis have lower white blood cell counts? Let $\alpha = 0.01$.

Sol'n:

1. Parameter: $\mu$ the mean white blood cell count

2. $H_0: \mu = 7250$

   $H_1: \mu < 7250$

3. Test Stat.:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4767 - 7250}{3204/\sqrt{15}} = -3.00$$

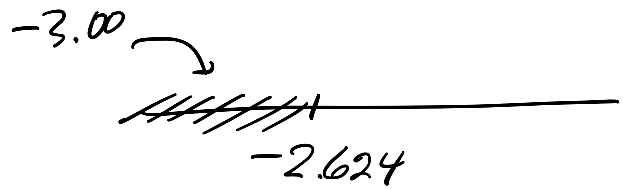
$\sigma$ unknown

4. Rejection Region:

Reject $H_0$ if $t < -t_\alpha$ ; $df = n - 1 = 14$.

$t_{14;\,0.01} = 2.624$ ;

Reject if $t < -2.624$.




5. Decision: Since $t = -3.00 < -2.624$, we reject $H_0$.

6. Conclusion: We are 99% Confident that the mean blood count of people infected with E-Canis is less than $7250/mm^3$.

Assumptions: (Case I) i) pop. is normal and ii) $n/N < 0.05$ ( i.e. independent obs'ns).

Ex. A car manufacturer wants to determine whether a luxury model gives Satisfactory mileage So that the overall EPA requirement on mileage per car manufactured can be met. The particular model will be Considered Satisfactory if the true mean mileage $\mu$ is greater than 20 miles per gallon. A field experiment was Conducted in which ten Cars were driven under almost identical Conditions and the miles per gallon were Computed for each. The results (in miles) were as follows.

23 18 22 19 19 22 18 18 24 22

Based on the data, is there sufficient evidence for the manufacturer to decide that the model is Satisfactory? Let $\alpha = 0.05$.

Sol'n:

1. Parameter: $\mu$, the mean mileage.

2. $H_0: \mu = 20$
   $H_1: \mu > 20$

3. Test statistic: $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{20.5 - 20}{2.32/\sqrt{10}} = 0.682$

Here, $\bar{x} = 20.5$, $S = 2.32$; $\sigma$ is unknown.

4. Rejection Region: Reject $H_0$ if $t > t_{0.05}$;

$d.f. = n - 1 = 10 - 1 = 9.$

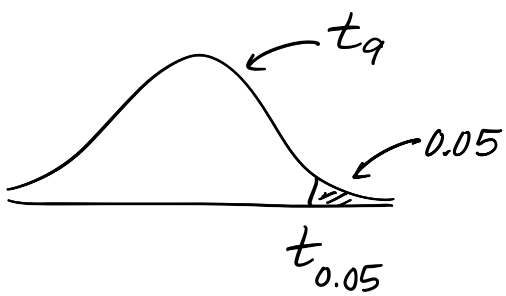



Table V:

| df | 0.05 |
|---|---|
| 9 | $\boxed{1.833}$ |

So, we reject $H_0$ if $t > 1.833$.

5. Decision: Since $t = 0.682 < 1.833$, we fail to reject $H_0$.

6. Conclusion: There is not enough evidence to suggest that the true mean mileage per gallon is greater than 20.