STAT-1302; Lecture 4; Jan. 18, '24

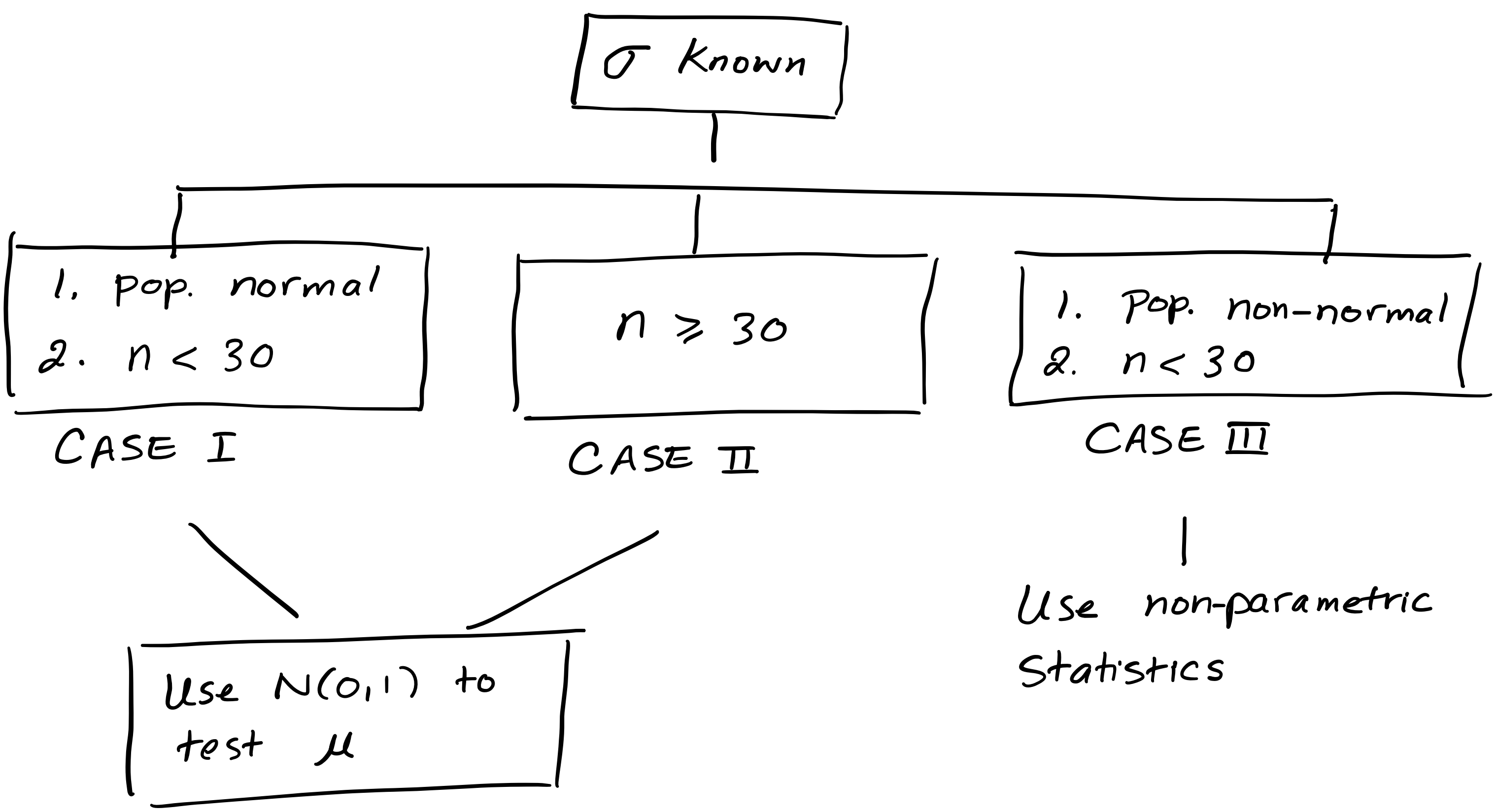
Power of a test:

$$\overset{\text{def}}{=} P(\text{ rejecting } H_0 \mid H_1 \text{ true })$$

$$= 1 - \beta$$

Power computations are useful for sample size determination.

§ 9.2 Hypothesis Tests about μ : σ known



Assumptions: In all three cases, we have a random sample $X_1, \dots, X_n$ and that $X_1, \dots, X_n$ are independent random variables (or $\frac{n}{N} < 0.05$).

Remark: N = Pop. size, n = sample size. Suppose the random sampling is done without replacement. If $\frac{n}{N} < 0.05$, then $X_1, \dots, X_n$ can be viewed as independent observations.

Test Statistic:

For Cases I and II, use

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

where

$\bar{x}$:= sample mean

μ_0 := the value of μ under H_0.

σ := pop. std. dev'n.

n := sample size

Under H_0, the Sampling distribution of

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \quad \text{is} \quad N(0,1).$$

Test (σ known)	Rejection Region
$H_0: \mu = \mu_0$ vs. $H_1: \mu > \mu_0$	$Z \geq z_\alpha$

$H_0: \mu = \mu_0$ vs. $H_1: \mu < \mu_0$	$Z \leq -z_\alpha$

$H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$	$\lvert Z \rvert > z_{\alpha/2}$

Recall: $\lvert x \rvert = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$\lvert -3 \rvert = -(-3) = 3 \quad ; \quad \lvert 5 \rvert = 5$

The above is the Rejection Region approach to testing $H_0: \mu = \mu_0$ (σ Known).

The p-value approach for this test is as follows.

<u>Test</u>	<u>P-value</u>
$H_0: \mu = \mu_0$, $H_1: \mu > \mu_0$	$P(Z > Z_{obs.})$
$H_0: \mu = \mu_0$, $H_1: \mu < \mu_0$	$P(Z < Z_{obs.})$
$H_0: \mu = \mu_0$, $H_1: \mu \neq \mu_0$	$P(\lvert Z \rvert > \lvert Z_{obs.} \rvert)$.

Ex. After taking a refresher course, a salesperson found that sales (in dollars) on nine random days were as follows:

1280 1250 990 1100 880 1300 1100 950 1050

Assume $\sigma = 100$. Has the refresher course had the desired effect, in that the mean sale is now more than \$1,000? Test at $\alpha = 0.01$. What is the p-value of the test?

1. Parameter : μ is the mean sale

2. $H_0: \mu = 1000$

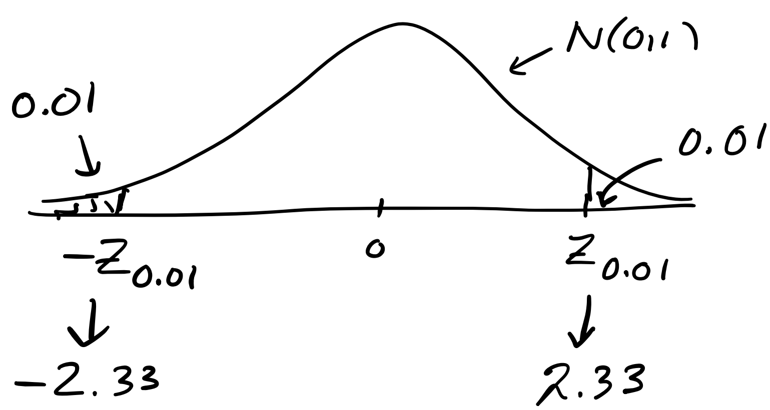
$H_1: \mu > 1000$

3. Test Statistic:

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{1100 - 1000}{100/\sqrt{9}} = 3$$

4. Reject if $Z > Z_\alpha$

$Z_{0.01} = ?$



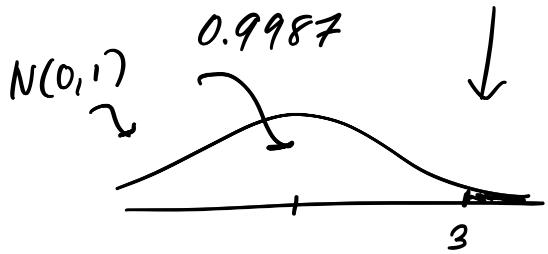
Reject H_0 if $Z > 2.33$.

5. Since $Z_{obs.} = 3 > 2.33$, we reject H_0.

$1-\alpha$; $\alpha = 0.01$

6. Conclusion: We are 99% Confident that the mean sale is now more than \$1,000.

$$\text{P-value} = P(Z > 3) = 1 - P(Z < 3)$$
$$= 1 - 0.9987$$
$$= 0.0013.$$



Aside: Since p-value $= 0.0013 < \alpha = 0.01$, we reject H_0. Same conclusion as before.

Ex. In advertising a brand of King-size Cigarettes, a tobacco Company says that the Customer Could Switch down to lower tar by buying their brand. A random Sample of 12 Cigarettes was selected from this brand and the Cigarettes were tested for their tar Content. The Sample mean tar Content was found to be 7.783 mg. Suppose it is reasonable to believe that the tar content for this brand is normally distributed with a standard deviation σ of 1 mg. At the 10% level, do the results of the study support the claim that the true mean tar content for this brand is less than 8 mg? What is the p-value?

1. Parameter: μ the mean tar content

2. $H_0: \mu = 8$

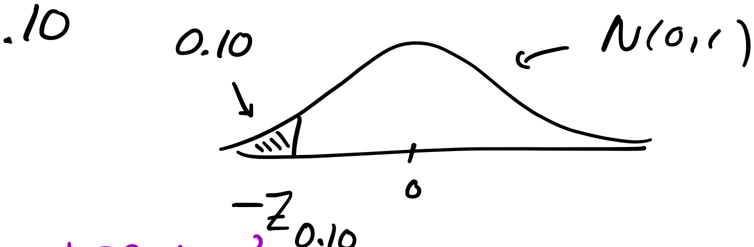
   $H_1: \mu < 8$

3. σ is known; test stat.

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{7.783 - 8}{1/\sqrt{12}} = -0.75$$

4. Reject H_0 if $Z < -Z_\alpha$

$\alpha = 0.10$



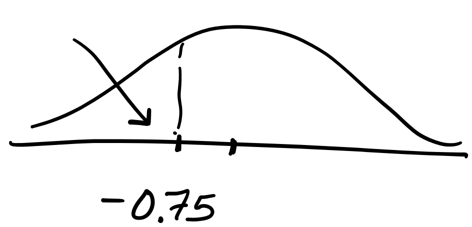
Reject H_0 if $Z < -1.28$.

5. Decision: Since $Z_{obs.} = -0.75 > Z_{0.10} = -1.28$, we fail to reject H_0.

6. Conclusion: There is not enough evidence to suggest that the mean tar content of this company's cigarettes is less than 8 mg.

$$p\text{-value} = P(Z < -0.75) = 0.2266$$

(From Table IV; App. B).



Since $p\text{-value} = 0.2266 > \alpha = 0.10$, we fail to reject H_0. Same conclusion as before.