STAT-1302; Lecture 3; Jan. 16, '24

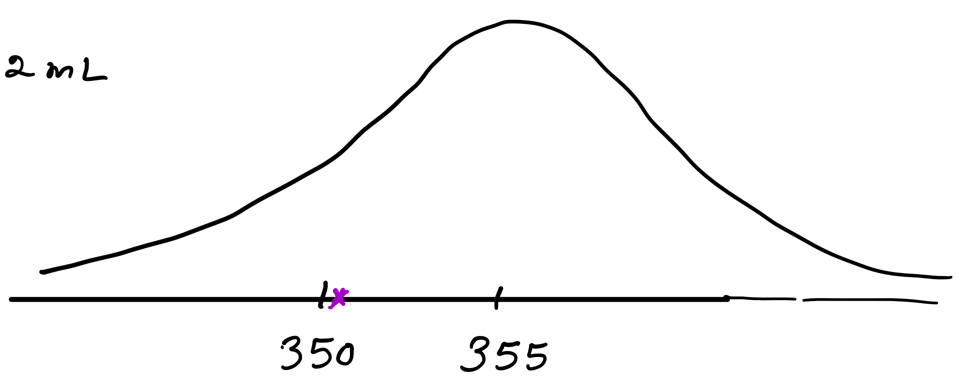
Motivating Example:

A quality control engineer draws a random sample of 100 cans of soft drink from an assembly line, and finds the average soda amount to be 350.2 mL. The cans are stated to contain 355 mL. Based on the sample mean can we conclude that on average the cans are being underfilled?

Scenario I: Let X = fill amount. $X \sim N(355, 5)$

$\bar{x} = 350.2$ mL

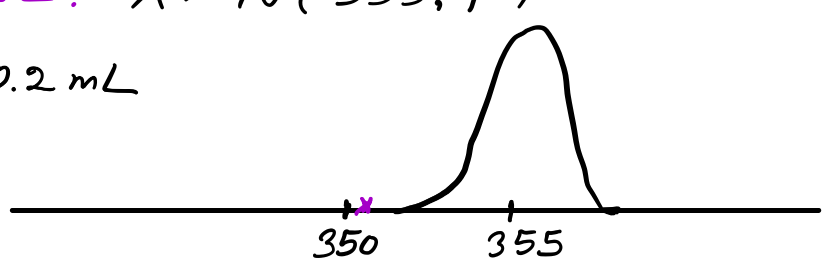
$n = 100$



Scenario II: $X \sim N(355, 1)$

$\bar{x} = 350.2$ mL

$n = 100$



Ch. 9 Hypothesis Testing.

Framework:

Steps for conducting a hypothesis test:

1. Identify the model parameter(s) of interest.

2. Write down the null hypothesis and the alternative hypothesis. These involve the parameter(s) of interest.

3. From the sample data calculate a test statistic (i.e. a single number computed from the random sample).

4. Based on the test statistic either

i) Check whether the test statistic falls into a rejection region

or

ii) Calculate the p-value of the test statistic.

5. Using 4), make a decision about whether there is enough evidence to reject the null hypothesis in favour of the alternative hypothesis.

6. Write a sentence about the conclusion in 5), in the words of the problem.

Ex. In the Soda example, the parameter of interest is μ, the mean fill amount.

Two Hypotheses:

Notation: H_0 ("H-null")

$\Leftrightarrow$ null hypothesis

H_1 or H_A $\Leftrightarrow$ alternative hypothesis

H_0: A statement about the parameter that is assumed to be true until it is declared false ($\Leftrightarrow$ the status-quo).

H_1 : A Statement about the parameter that is the complement of H_0.

Ex. μ = mean fill amount of Cans

$H_0: \mu = 355$ (In fact, $H_0: \mu \geq 355$).

$H_1: \mu < 355$ (In this course, always write $H_0: \mu = \dots\dots$)

Remark : Choice for H_1 are " $<$ ", " $>$ ", or " $\neq$ ".

H_1 may be one-sided or two-sided.

Ex. $H_0: \mu = 355$

$H_1: \mu < 355$

one-sided (left-tailed)

or

$H_0: \mu = 355$

$H_1: \mu > 355$

one-sided (right-tailed)

$H_0: \mu = 355$ two-sided / two-tailed test.

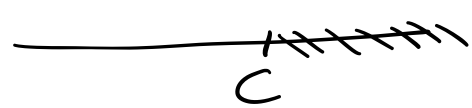
$H_1: \mu \neq 355$

One way to make a decision about whether or not to reject H_0 in favour of H_1 is to construct a rejection region using critical value. So that if our test statistic falls into the rejection region, we reject H_0 in favour of H_a.

Ex: $H_0: \mu = 355$

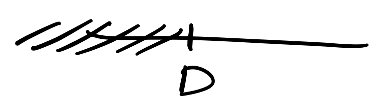
$H_1: \mu > 355$

Rej. Region $= \{ T : T > C \}$



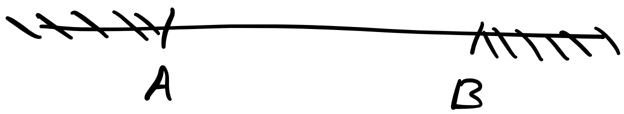
Ex: $H_0: \mu = 355$

vs. $H_1: \mu < 355$



Rej. Region: $\{ T : T < D \}$

$H_0 : \mu = 355$ vs. $H_1 : \mu \neq 355$



The critical value is based on α (level of significance).

		Truth	
		H_0 true	H_1 true
Decision	Reject H_0	Type I Error	✓
	Accept H_0	✓	Type II Error

Type I Error := Reject H_0 when H_0 is true

Type II Error := Accept H_0 when H_1 is true.

$\alpha := P(\text{Type I Error})$

$\beta := P(\text{Type II Error})$

The researcher selects α prior to looking at the data. Small α values are selected:

$\alpha = 0.01$, $\alpha = 0.05$, $\alpha = 0.10$.

Ideally, we would like to control α and β, simultaneously. But this is not possible. Convention: Type I Error is viewed as more serious. $\therefore$ Select α only.

Interpretation:

Suppose we select $\alpha = 0.05$. Since α is a probability, based on the relative frequency interpretation of probability, if we were to conduct 100 tests, we would expect 5 of those tests (i.e. 5%) to reject H_0 when H_0 is true. Or, 95 out of 100 tests are expected to make a correct decision about rejecting H_0. Our particular test, may be one of the

5% incorrect decisions, or it may be one of the 95% Correct decisions.

Ex. Cola example:

H_0: $\mu = 355$

Type I Error: Stating $\mu < 355$ when in fact $\mu = 355$; or Stating $\mu > 355$ when in fact $\mu = 355$; or stating $\mu \neq 355$ when in fact $\mu = 355$.

Type II Error:

i) Stating $\mu = 355$ when in fact $\mu < 355$;

or ii) Stating $\mu = 355$ when in fact $\mu > 355$;

or iii) Stating $\mu = 355$ when in fact $\mu \neq 355$.

$$\left.\begin{array}{l} \text{ART} \\ A \to \alpha \\ R \to \text{Reject} \\ T \to \text{True} \end{array}\right\} \text{ in terms of } H_0$$

BAF $\rightarrow$ False
$\downarrow$ $\searrow$
β Accept

} in terms of H_0

$\beta = P(\text{Type II Error})$

$= P(\text{Accept } H_0 \text{ when } H_0 \text{ false})$

$= P(\text{accept } H_0 \text{ when } H_1 \text{ true})$

P-value: The probability of getting the data you got (Summarized by the test statistic) or Something more extreme assuming H_0 is true.

*P-value interpretation:

Small p-values lead to rejection of H_0. (i.e. p-value $< \alpha$), because this says that the data are not very likely if H_0 is true.