STAT-1302 ; Lecture 1 ; Jan. 9, '24

Ch. 8 Review :

i) Confidence interval (CI) for a pop. mean, μ, when pop. std. deviation, σ, is known.

ii) CI for μ when σ is unknown.

iii) CI for popul'n proportion, p, in large samples.

Informal def'n of a CI :

A range of plausible values for a pop. parameter based on a given confidence level.

Formal def'n of a CI :

A $100(1-\alpha)\%$ CI for a parameter represents the following: Upon repeated sampling and construction of a CI for the parameter each time, we expect to see $100(1-\alpha)\%$ of the intervals Containing the parameter.

$100(1-\alpha)\%$ CI for μ when σ is known:

Result: Let $X_1, \ldots, X_n$ be a random Sample from a population with mean μ and Std. dev'n σ.

Then

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

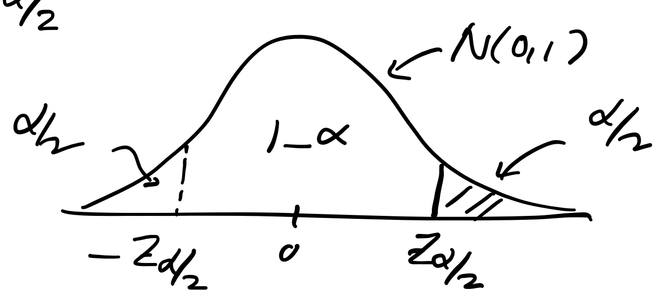
is exactly normal if the pop. is normal.

In particular, $Z \sim N(0,1)$.

If the pop. is not normal but $n > 30$, the $Z \approx N(0,1)$ (approx. distrib'n).

(by the Central Limit Th'm).

Recall: $z_{\alpha/2}$



$$1-\alpha = P\left(-Z_{\alpha/2} < Z < Z_{\alpha/2}\right)$$

$$= P\left(-Z_{\alpha/2} < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < Z_{\alpha/2}\right)$$

$$= P\left(\bar{x} - Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right).$$

$\therefore$ A $100(1-\alpha)\%$ CI for μ when σ is known is given by

$$\bar{x} \pm Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$

$\bar{x}$ = Sample mean

$Z_{\alpha/2}$ = Critical value

σ = pop. Std. dev'n

n = Sample Size

σ Known

CASE I

1. pop. normal
2. $n < 30$

CASE II

$n > 30$

CASE III

1. pop. not normal
2. $n < 30$

Use
$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

Use non-parametric statistics

ex. A publishing Company has just published a new textbook. The Company wants to estimate the average price of all books similar to its textbook. The research department took a random sample of 25 textbooks and found the sample mean price to $90.50. The standard deviation of the price of all such textbooks is $7.50 ($\sigma$), and the pop. of such prices is normal. Construct a 90%

Confidence interval for the mean price of all such textbooks.

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Given: X = price, $X \sim N(\mu, 7.50)$.

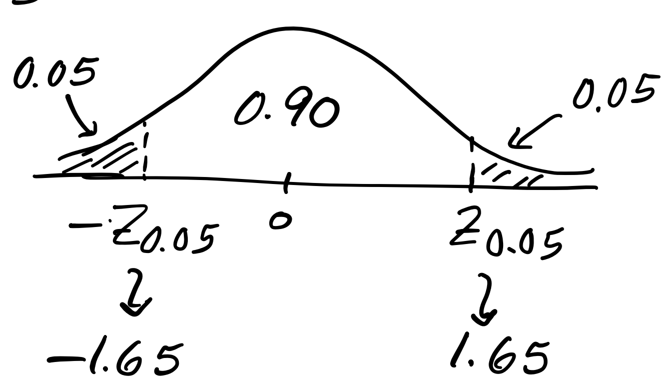
$\bar{x} = 90.50$; $n = 25$ (Case I).

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 90.50 \pm 1.65 \cdot \frac{7.50}{\sqrt{25}} = (*)$$

$100(1-\alpha)\% = 90\% \Rightarrow 1-\alpha = 0.9$

$\alpha = 0.1 \Rightarrow \alpha/2 = 0.05$

$z_{\alpha/2} = z_{0.05}$



margin of error

$(*) = 90.5 \pm 2.48 = (\$88.02, \$92.98)$.

Interpretation: We are 90% Confident that the mean price of all such textbooks is between \$ 88.02 and \$92.98.

A 95% CI for μ here?

$Z_{\alpha/2} = ?$

$100(1-\alpha)\% = 95\%$, $1-\alpha = 0.95$; $\alpha = 0.05$,

$\alpha/2 = 0.025$, $Z_{0.025} = 1.96$.

A 95% CI for μ is

$$\bar{x} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 90.5 \pm 1.96 \cdot \frac{7.5}{\sqrt{25}}$$

$$= 90.5 \pm 2.94$$

$$= (\$87.56, \ \$93.44).$$

Interpretation: We are 95% Confident that the true mean price of all such textbooks is between \$87.56 and \$93.44.

Remark: With a large confidence level (e.g. 95%), we are more likely to capture the true mean μ but the cost is that the CI for μ is wider than the 90% CI for μ.

Summary:

1. Need to compute CIs
2. Interpretation.
3. Assumptions (Case I, II).