

# Chapter 3: Section 3-6

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- **Result:** Among **200,745** children injected with the Salk vaccine, **33** developed poliomyelitis. i.e.

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### Risks & Odds

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- Can't interpret this probability unless we know "P(polio|Placebo)". (Placebo means no vaccine).

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## Prospective Study of Polio and the Salk Vaccine

	Polio	No Polio	<b>Total</b>
Salk Vaccine	33	200,712	200,745
Placebo	115	201,114	201,229

- **Polio rate for treatment group:**

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## Prospective Study of Polio and the Salk Vaccine

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### Prospective Study of Polio and the Salk Vaccine

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- An informal comparison suggests differences between the two rates.

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Table 3-4: Generalized Table Summarizing Results of a Prospective Study

	Disease	No Disease
Treatment (or Exposed)	a	b
Placebo (or Not Exposed)	c	d

Comparison will be made using the following risk measures:

- (i) **absolute risk reduction**,
- (ii) **relative risk/risk ratio (RR)**,
- (iii) **number needed to treat (NNT)**,
- (iv) **odds ratio (OR)/relative odds**



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### Summarizing Results of a Prospective Study

#### Definition: **absolute risk reduction**

$$\begin{aligned}\text{absolute risk reduction} &= |P(\text{event occurring in treatment group}) \\ &\quad - P(\text{event occurring in control group})| \\ &= |p_t - p_c| \\ &= \left| \frac{a}{a+b} - \frac{c}{c+d} \right|\end{aligned}$$

“Treatment” can be replaced by “the presence of some condition”, or “exposed to disease”, or some other equivalent description.

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### Absolute Risk Reduction for Salk Vaccine

$$\begin{aligned}\text{absolute risk reduction} &= |P(\text{polio}|\text{Salk vaccine}) - P(\text{polio}|\text{placebo})| \\ &= \left| \frac{33}{33 + 200,712} - \frac{115}{115 + 201,114} \right| \\ &= 0.000407\end{aligned}$$

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### Relative Risk/ Risk Ratio (RR) in a Prospective Study

#### Notation

$p_t$  = proportion (or incidence rate) of some characteristic in a *treatment group*

$p_c$  = proportion (or incidence rate) of some characteristic in a *control group*

#### Definition: RR

$$RR = \frac{p_t}{p_c} = \frac{\frac{a}{a+b}}{\frac{c}{c+d}}$$

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### Relative Risk/ Risk Ratio (RR) for Salk Vaccine

$$p_t = \text{proportion (or incidence rate) of some characteristic in a} \\ \textit{treatment group} = \frac{33}{33 + 200,712} = 0.000164$$

$$p_c = \text{proportion (or incidence rate) of some characteristic in a} \\ \textit{control group} = \frac{115}{115 + 201,114} = 0.000571.$$

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### Relative Risk/ Risk Ratio (RR) for Salk Vaccine

$$p_t = \text{proportion (or incidence rate) of some characteristic in a treatment group} = \frac{33}{33 + 200,712} = 0.000164$$

$$p_c = \text{proportion (or incidence rate) of some characteristic in a control group} = \frac{115}{115 + 201,114} = 0.000571.$$

$$RR = \frac{p_t}{p_c} = \frac{0.000164}{0.000571} = 0.287$$

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### Relative Risk/ Risk Ratio (RR) for Salk Vaccine

#### Interpretation:

- The polio rate for children given the Salk vaccine is 0.287 of the polio rate for children given a placebo.

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- The polio rate for children given the Salk vaccine is 0.287 of the polio rate for children given a placebo.
- Note:  $RR < 1$  means treatment results in reduced risk.  $RR = 1$  means no difference between treatment and placebo.

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### Relative Risk/ Risk Ratio (RR) for Salk Vaccine

#### Interpretation:

- The polio rate for children given the Salk vaccine is 0.287 of the polio rate for children given a placebo.
- Note:  $RR < 1$  means treatment results in reduced risk.  $RR = 1$  means no difference between treatment and placebo.
- Using  $1/0.287 = 3.48$  means that children in the placebo group are 3.48 times more likely to get polio.



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### Number Need to Treat (NNT)

Relative risk can be misleading. Read p. 130 of the textbook.

#### Definition

The **number needed to treat** is the number of subjects that must be treated in order to prevent *one* event, such as a disease or adverse reaction.

$$\begin{aligned} \text{NNT} &= \frac{1}{\text{absolute risk reduction}} \\ &= \frac{1}{\left| \frac{a}{a+b} - \frac{c}{c+d} \right|} \end{aligned}$$

(rounded *up* to the next larger whole number)

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### Number Need to Treat (NNT) for Salk Vaccine

$$p_t = 0.000164$$

$$p_c = 0.000571$$

$$\text{absolute risk reduction} = |0.000164 - 0.000571| = 0.000407$$

$$\begin{aligned} \text{NNT} &= \frac{1}{\text{absolute risk reduction}} \\ &= \frac{1}{0.000407} = 2457.002457 \approx 2458 \end{aligned}$$

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### Number Need to Treat (NNT) for Salk Vaccine

#### Interpretation:

We would need to vaccinate 2458 children with the Salk vaccine (instead of a placebo) to prevent *one* of the children from getting polio.

Next, we study a risk measure that may be used in a *retrospective* or *prospective* study.

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### Odds

#### Definition

The **actual odds against** event  $A$  occurring are the ratio  $P(\bar{A}) / P(A)$ , expressed in the form  $m : n$  (or “m to n”), where  $m$  and  $n$  are integers having no common factors.

#### Definition

The **actual odds in favor** event  $A$  occurring are the ratio  $P(A) / P(\bar{A})$ . If the odds against  $A$  are  $m : n$ , then the odds in favor of  $A$  are  $n : m$ .

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### Retrospective Study of Newborn Discharge & Rehospitalization Example

	Rehospitalized	Not rehospitalized	<b>Total</b>
Early discharge	457	3199	3656
Late discharge	260	2860	3120

- (a) For those babies who were discharged early, find the probability of being rehospitalized within a week.
- (b) For those babies who were discharged early, find the odds in favor of being rehospitalized early.

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### Retrospective Study of Newborn Discharge & Rehospitalization Example

$$(a) P(\text{rehospitalized} | \text{Early discharge}) = \frac{457}{3656} = \frac{1}{8}.$$

(b) Odds in favor of re hosp. for early discharge group:

$$\frac{P(\text{rehospitalized} | \text{Early discharge})}{P(\overline{\text{rehospitalized}} | \text{Early discharge})} = \frac{1/8}{7/8} = \frac{1}{7}.$$

#### Interpretation:

With odds of 1:7 in favor of rehospitalization for babies discharged early, the odds *against* rehospitalization for early discharge are 7:1.

How does the likelihood of re hosp. differ between early discharge group and late discharge group? Use *odds ratio*.

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### Odds Ratio (OR)/Relative Odds

#### Definition:

In a *retrospective* or *prospective study*, the **odds ratio** (or **OR** or **relative odds**) is a measure of risk found from the ratio of the odds for the treatment group (or case group exposed to the risk factor) to the odds for the control group, evaluated as follows.

$$\begin{aligned} \text{OR} &= \frac{\text{odds in favor of event for treatment (or exposed group)}}{\text{odds in favor of event for control group}} \\ &= \frac{ad}{bc} \end{aligned}$$

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### Computing Odds Ratio for Rehospitalization Example

$$\text{OR} = \frac{\text{odds for rehospitalization in early discharge group}}{\text{odds for rehospitalization in late discharge group}}$$



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$$\begin{aligned}\text{OR} &= \frac{\text{odds for rehospitalization in early discharge group}}{\text{odds for rehospitalization in late discharge group}} \\ &= \frac{457/3199}{260/2860} = \frac{1/7}{1/11} = 1.571 =\end{aligned}$$

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### Computing Odds Ratio for Rehospitalization Example

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#### Interpretation:

The odds in favor of rehospitalization are 1.571 times higher for babies discharged early when compared to those discharged late. That is, newborns discharged early are at substantially increased risk of rehospitalization.

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### Odds Ratio versus Relative Risk: A Caution

- **Prospective Study:** Use RR and/or OR.
- **Retrospective Study:** Use OR *only*.
- Read p. 133-134.