Objectives:

- Poisson Distribution and its moments: Section 5-6 of text.

Figure: Siméon-Denis Poisson: 21 June 1781– 25 April 1840

Chapter 5: Section 5-6

The Poisson Distribution

Definition

The **Poisson distribution** is a discrete probability distribution that applies to occurrences of some event over a specified interval. The interval can be time, distance, area, volume, or some similar unit.

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Examples:

- The number of daily covid-19 cases in Winnipeg
- The number of mutations in set sized regions of a chromosome
- The number of dolphin pod sightings along a flight path through a region
- The number of particles emitted by a radioactive source in a given time
- The number of births per hour during a given day

Refer to previous page examples.

- In such situations we are often interested in whether the events occur randomly in time or space.
- The distribution of counts is useful in uncovering whether the events might occur randomly or non-randomly in time (or space).
- Simply looking at the histogram isn't sufficient if we want to ask the question whether the events occur randomly or not.
- To answer this question we need a probability model for the distribution of counts of random events that dictates the type of distributions we should expect to see.

Conditions to apply the Poisson probability distribution to a random variable X.

- The random variable, X, is a discrete random variable. (It counts the number of occurrences in an interval.)
- The occurrences are random.
- The occurrences are independent.

Poisson Probability Distribution Formula

$$P(x) = \frac{e^{-\lambda}.\lambda^{x}}{x!}, x = 0, 1, 2, \ldots$$

λ is the mean number of occurrences in the interval of interest.

$e \approx 2.7183$.

This expression is provided for the final exam.

Let X be the random variable representing the number of occurrences in a unit time or interval satisfying the Poisson distribution postulates.

Mean and Standard Deviation; Notation

- The mean is λ.
- The variance is λ.
- The standard deviation is $\sigma = \sqrt{\lambda}$.
- Notation: Write

$$X \sim \textit{Poisson}(\lambda).$$

Example Poisson-1

2.3 patients arrive at a hospital's emergency room on Fridays between 10:00 p.m. and 11:00 p.m. Define X to be the random variable that counts the number of patients in the 10-11 p.m. slot.

(a) What is the probability that exactly four patients arrive between 10:00 p.m. and 11:00 p.m. on a typical Friday?

(b) Find the mean and standard deviation of the number of patients that arrive between 10:00 p.m. and 11:00 p.m. on a typical Friday?

Example Poisson-1 continued:

- $P(4) = e^{-2.3}\frac{2.3^4}{4!} = 0.12.$

- Mean: $\lambda = 2.3$; Standard deviation: $\sigma = \sqrt{2.3} = 1.52$

Example Poisson-2

The U.S. Centres for Disease Control reports **7.7** cases of typhoid fever per week, on average from all over the United States. Define X to be the random variable that counts the number of typhoid cases per week.

(a) What is the probability of at least one typhoid case per week?

(b) Find the mean and standard deviation of the number of typhoid cases per week.

Example Poisson-2 continued:

- $P(X \geq 1) = 1 - P(X < 1) = 1 - P(0) = 1 - e^{-7.7} = 1 - 0.000453 = 0.9995.$

- Mean: $\lambda = 7.7$; Standard deviation: $\sigma = \sqrt{7.7} = 2.77$

Example Poisson-3

For a recent period of **100** years, there were **93** major earthquakes in the world. Find the mean number of earthquakes per year. Suggest a suitable probability model for the number of earthquakes per year. What is the probability of no earthquakes in a randomly selected year?

Example Poisson-3 continued:

- Put $\lambda = 93/100 = 0.93$ as the rate parameter.
- Then, $X \sim \mathit{Poisson}(0.93)$.
- $P(0) = e^{-0.93} = 0.39$.

Example Poisson-4

Suppose that the average number of zooplankten per litre of lake water is **2.** What is the probability that **5** zooplankten will be found in a random sample of **3** litres of lake water?

Example Poisson-4 continued:

- Let X be the number of zooplankten per litre. Then $X \sim Poisson(2)$.

- Put Y to be the number of zooplankten per 3 litres. Then, $Y \sim Poisson(6)$. i.e. the mean of Y is 3×2.

- Now find $P(5) = e^{-6}\dfrac{6^5}{5!} = 0.161$.

Binomial vs. Poisson

- Binomial: counts the number of *successes* in *n* independent Bernoulli trials.

- Poisson: counts the number of *successes* in an *interval*.

Examples of Applications of Poisson Distribution

- Ontario Lottery Retailer scandal.
  http://probability.ca/jeff/ftpdir/lotteryartref.pdf