Problem 1

The sensitivity of a screening test is **0.95** and its specificity is **0.85**. The rate of the disease for which the test is used is **0.002**. What is the positive predictive value of the test?

Problem 1 Solution

$$P(D|+) = \frac{0.95 \times 0.002}{0.95 \times 0.002 + (1 - 0.85) \times (1 - 0.002)} = 0.0125$$

Problem 2

A shop has **11** video games to choose from, and **4** of them contain extreme violence. A customer picks **3** of these 11 games at random. What is the probability that the number of extremely violent games among the three selected games is exactly two?

Problem 2 Solution

$$\frac{\binom{4}{2} \times \binom{7}{1}}{\binom{11}{3}} = \frac{6 \times 7}{165} = 0.255$$

Problem 3

The West African country of Guinea has the highest rate of malaria in the world, with 75% of its population infected. A physician sees **6** patients in a given hour. Assuming that patient visits are independent, what is the probability that all of them are infected? What is the probability that all but 1 of these 6 patients are infected with malaria?

Problem 3 Solution

Let X be the number of patients who are infected with malaria out of 6 patients. Then $X \sim Bin(6, 0.75)$.

- $p(6) = \binom{6}{6} 0.75^6 0.25^0 = 0.178.$
- $p(5) = \binom{6}{5} 0.75^5 0.25 = 0.356$

Problem 4

About **80%** of unvaccinated children who are exposed to whooping cough (pertusis) will develop an infection, as opposed to only about **5%** of vaccinated children. A group of **20** children at a nursery school are exposed to whooping cough by playing with an infected child.

(a) If all 20 have been vaccinated, what are the mean and standard deviation of new infections?

(b) If none of the 20 have been vaccinated, what are the mean and standard deviation of new infections?

Problem 4 Solution

(a) Let X be the number of infected children among 20 vaccinated children. Then $Y \sim Bin(20, 0.05)$. The mean and standard deviation are $\mu_X = E(X) = 20 \times 0.05 = 1$ and $\sigma_X = \sqrt{20 \times 0.05 \times 0.95} = 0.975$.

(b) Let Y be the number of infected children among 20 unvaccinated children. Then $Y \sim Bin(20, 0.8)$. The mean and standard deviation are $\mu_Y = E(Y) = 20 \times 0.8 = 16$ and $\sigma_Y = \sqrt{20 \times 0.8 \times 0.2} = 1.789$.

Problem 5

The mean number of daily surgeries at a local hospital is 6.2. If we assume that surgeries are random, independent events, what is the probability that there would be only two or fewer surgeries in a given day?

Problem 5 Solution

Let X be the number of surgeries per day at a local hospital. Then $X \sim Poisson(6.2)$. We have $P(X \leq 2) = p(0) + p(1) + p(2) = 0.0536$ because of the following:

(i) $p(0) = e^{-6.2} = 0.00203$

(ii) $p(1) = e^{-6.2} 6.2 = 0.0126$

(iii) $p(2) = e^{-6.2} \frac{6.2^2}{2!} = 0.0390.$

Problem 6

A health clinic gets an average of 4.5 pediatric appointments per day for routine vaccination. The appointments are unrelated and so we can assume that they are random, independent events. Let X be the count of pediatric appointments per day for routine vaccination. What distribution does X follow, approximately? Give the mean and standard deviation of X.

Problem 6 Solution

Let X be the number of appointments per day. $X \sim Poisson(4.5)$. Then, $\mu = E(X) = 4.5$ is the mean and $\sigma = \sqrt{Var(X)} = \sqrt{4.5} = 2.12$ is the standard deviation of the number of appointments per day.

Problem 7

Let the random variable X denote the number of days of a patient's stay in the intensive-care unit of a hospital after a particular operation. Considering the population of all patients, suppose that the following probability distribution is assessed for X:

x	1	2	3
P(x)	0.3	0.4	0.3

(a) Find the mean of X.

(b) Find the standard deviation of X.

Problem 7 Solution

- $\mu = \sum x.p(x) = 1(0.3) + 2(0.4) + 3(0.3) = 2.$
- $Var(X) = \sum x^2.p(x) - \mu^2 = 1^2(0.3) + 2^2(0.4) + 3^2(0.3) - 2^2 = 0.6$. Then, $\sigma = \sqrt{Var(X)} = \sqrt{0.6} = 0.775.$

Problem 8

An aptitude test administered to aircraft pilot trainees requires a series of operations to be performed in quick succession. Suppose the time needed to complete the test is normally distributed with mean $\mu = \mathbf{90}$ minutes and standard deviation $\sigma = \mathbf{20}$ minutes.

(a) To pass the test, a candidate must complete it within **80** minutes. What percentage of the candidates will pass the test?

(b) If the top **2.5**% of the candidates are to be given a certificate of commendation, how fast must a candidate complete the test to be eligible for a certificate?

Problem 8 Solution

Let X be the time needed to complete the test. Then $X \sim N(90, 20)$.

(a) $P(X < 80) = P(Z < \frac{80-90}{20}) = P(Z < -0.5) = 0.308$. i.e. about 31% are expected to pass.

(b) We seek the 2.5th percentile of the normal distribution with mean 90 and standard deviation 20; call this number q. The 2.5th percentile of the standard normal distribution is -1.96. Equate the z-score of q to -1.96 and solve for q. That is,

$$\frac{q-90}{20} = -1.96.$$

Therefore $q = -1.96(20) + 90 = 50.8$. Candidates must complete the test within approximately 51 minutes to be given a certificate.