# Section 3.8: Counting

Consider the following set-up.

- There may be too many outcomes in the sample space for us to list.
- Similarly, the number of outcomes of interest in the sample space may also be large.
- However, there are rules for counting the number of items in the sample space and the frequency of the outcome of interest in the sample space.
- Then, we may apply rule 1 or rule 2, and calculate the probability of the outcome of interest using its relative frequency in the sample space.

# Motivating Example

A biology professor gives a surprise quiz consisting of 10 true/false questions, and she states that passing requires at least 7 correct responses. Assume that an unprepared student adopts the questionable strategy of guessing for each answer.

(a) Find the probability that the first 7 responses are correct and the last 3 are wrong.

(b) Is the probability from part (a) equal to the probability of passing? Why or why not?

# Fundamental Counting Rule

For a sequence of two events in which the first event can occur in $m$ ways and the second event can occur $n$ ways, the events together can occur a total of $m \times n$ ways.

- Note: Generalizes to more than two events.

# Example 1

In designing a computer, if a *byte* is defined to be a sequence of 8 bits and each bit must be a 0 or a 1, how many different bytes are possible?

# Example 2

At a restaurant, customers can select one meat dish, one vegetable, one beverage and one dessert. The menu offers two meat dishes, three vegetables, three beverages, and one dessert. How many different meals are possible?

Notation The factorial symbol ! denotes the product of decreasing positive whole numbers. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$. By definition, $0! = 1$.

# Factorial Rule

A collection of $n$ different items can be arranged in order $n!$ different ways. This **factorial rule** reflects the fact that the first item may be selected $n$ different ways, the second item may be selected $n - 1$ ways, and so on.

# Example 3

You have just started your own medical transport company and you have one plane for a route connecting Austin, Boise, and Chicago. One route is Austin-Boise-Chicago and a second route is Chicago-Boise-Austin.

(a) How many total routes are possible?

(b) How many different routes are possible if service is expanded to include a total of 8 cities?

# Permutations Rules:

(When the items are all different) The number of **permutations** (or sequences) of $r$ items selected from $n$ available items (without replacement) is

$$_nP_r = \frac{n!}{(n-r)!}$$

(When Some Items Are Identical to Others) If there $n$ items with $n_1$ alike, ..., $n_k$ alike, the number of **permutations** (or sequences) of all $n$ items is

$$\frac{n!}{n_1!n_2!\dots n_k!}$$

# Example 4

Five lifeguards are available for duty on Saturday afternoon. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations?

# Example 5: Gender Sequences

(a) A couple plans to have eight children. How many different gender sequences are possible?
(b) If a couple plans to have four boys and four girls, how many different gender sequences are possible?
(c) What is the probability that when a couple has eight children, the result will consist of four boys and four girls?

# Example 6

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(b) Two of the committee members are Ellen and Jose. Assume the assignments are made at random. What is the probability that Jose is president and Ellen is vice-president?

# Example 6

(a) A committee of eight people must choose a president, a vice-president, and a secretary. In how many ways can this be done?

(b) Two of the committee members are Ellen and Jose. Assume the assignments are made at random. What is the probability that Jose is president and Ellen is vice-president?

(c) What is the probability that either Ellen or Jose is president and the other is vice-president?

# Example 7

- Sometimes, we want to count the number of permutations of a part of a group.

Ten runners enter a race. The first-place finisher will win a gold medal, the second-place finisher will win a silver medal, and the third-place finisher will win a bronze medal. In how many different ways can the medals be awarded?

# Example 8

A biology professor gives a surprise quiz consisting of 10 true/false questions, and she states that passing requires at least 7 correct responses. Assume that an unprepared student adopts the questionable strategy of guessing for each answer. Find the probability the probability of passing?

# Combinations Rule

The number of **combinations** of $r$ items selected from $n$ different items is

$$_nC_r = \frac{n!}{(n-r)!r!}$$

- A total of $n$ *different* items are available.
- We must select $r$ of the $n$ items *without replacement*.
- We must consider rearrangements of the same items to be the same. e.g. The combination “ABC” is the same as “CBA”.

# Example 9

A batch of pills consists of 7 that are good and 3 that are defective. Three pills are randomly selected and tested.

(a) How many different samples of pills are possible when three pills are randomly selected (without replacement) from the 10 that are available?

(b) Find the probability that all three defective pills are selected.

# Example 10

A carton contains **12** eggs, **3** of which are cracked. If we randomly select **5** of the eggs for hard boiling, what is the probability of the following events?

(a) All of the cracked eggs are selected.

(b) None of the cracked eggs are selected.

(c) Two of the cracked eggs are selected.