Checking Independence of Events:

Example A

Suppose two doctors A and B test all patients coming into a clinic for syphilis. Let A^+ be the event "doctor A makes a positive diagnosis", and B^+ be the event "doctor B makes a positive diagnosis". Suppose doctor A diagnoses 10% of all patients as positive, doctor B diagnoses 17% of all patients as positive, and both doctors diagnose 8% of all patients as positive. Are the events A^+ and B^+ independent?

Checking Independence of Events:

Example B

The following table shows results from the 2002 U.S. General Social Survey for married adults classified according to gender and level of happiness.

	Very Happy	Pretty Happy	Not too Happy	Total
Male	221	95	9	325
Female	149	120	9	278
	370	215	18	603

1. What is the probability that a married adult is very happy given that the adult's gender is male?
2. For these subjects are the events being very happy and being male independent?

Bayes' Theorem:

Screening Tests

- Clinicians often use the results of screening tests to evaluate the presence/absence of a disease. e.g. Blood test for Downs' Syndrome, PSA test for prostate cancer.
- But screening tests are not always infallible; they may result in *false positives* or *false negatives*.
- A *false positive* results when a test indicates a positive status when the true status is negative.
- A *false negative* results when a test indicates a negative status when the true status is positive.

Screening Tests Cont'd:

The following questions must be answered in order to evaluate the usefulness of test results and symptom status in determining whether or not a subject has some disease:

1. Given that a subject has the disease, what is the probability of a positive test result (or the presence of a symptom)?
2. Given that a subject does not have the disease, what is the probability of a negative test result (or the absence of a symptom)?
3. Given a positive screening test (or the presence of a symptom), what is the probability that the subject has the disease?
4. Given a negative screening test result (or the absence of a symptom), what is the probability that the subject does not have the disease?

Screening Tests Cont'd:

Definitions

- The **sensitivity** of a test (or symptom) is the probability of a positive test result (or presence of the symptom) given the presence of the disease; $P(+|D)$.
- The **specificity** of a test (or symptom) is the probability of a negative test result (or absence of the symptom) given the absence of the disease; $P(-|\overline{D})$.
- The **positive predictive value** of a screening test (or symptom) is the probability that a subject has the disease given that the subject has a positive screening test result (or has the symptom); $P(D|+)$.
- The **negative predictive value** of a screening test (or symptom) is the probability that a subject does not have the disease, given that the subject has a negative screening test result (or does not have the symptom); $P(\overline{D}|-)$.

Bayes' Theorem

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B|A) \times P(A) + P(B|\overline{A}) \times P(\overline{A})}$$

Rev. Thomas Bayes: 1702-1761

Bio: `https://bayesian.org/bayes`

Example 1

Suppose one tenth of one percent of the population has a disease. Suppose that a diagnostic test for this disease exists and has 95% reliability. That is, 95% of the time, the disease status of diseased and non-diseased patients is correctly identified. A person is randomly chosen from the population and has a positive test result. What is the probability that this person has the disease?

Example 2

The U.S. National Cancer Institute estimates that 3.65% of women in the sixties get breast cancer. Of those of who have cancer, a mammogram can typically identify 85% of cases correctly. Of those without cancer, 95% are correctly identified to be free of cancer. What is the PPV of a mammogram?

Example 3

A study of the PSA blood test for diagnosing prostate cancer in men used a sample of 2620 men who were 40 years or older. When a positive test result was identified as a PSA test reading of at least 4, the sensitivity of the test was estimated to be 0.86, but the specificity only 0.33. Suppose that only 10% of those who take the PSA truly had prostate cancer. Find the PPV and NPV of this test.