Statistical Analysis I (STAT-1301)
Practice Problems with Solutions
Chapter 6

[Question 1]

A nurse supervisor has found that staff nurses complete a certain task with a mean of 10 minutes and a standard deviation of 2 minutes. The times required to complete the task are approximately normally distributed.

a) What is the median of the distribution?
- In a normal distribution mean=median, so median=10.

b) Find the probability that a nurse completes the task in less than 4 minutes.
- Let X be the random variable that represents the time required to complete the task. Therefore, we have that $X \sim N(10, 2)$.
- $z = \frac{x-\mu}{\sigma} = \frac{4-10}{2} = -3$.
- $P(X < 4) = P(Z < -3) = 0.0013$.

c) Find the probability that a nurse requires more than 11 minutes to complete the task.
- $z = \frac{x-\mu}{\sigma} = \frac{11-10}{2} = 0.5$.
- $P(X > 11) = P(Z > 0.5) = 1 - P(Z \leq 0.5) = 1 - 0.6915 = 0.3085$.

d) The nurse supervisor plans to set up observation conditions for the slowest 5 % nurses. What time should the supervisor nurse choose for the cut off separating the slowest 5% from the others?
- We must find x such that $P(X > x) = 0.05$.
- $P(X \leq x) = 1 - P(X > x) = 1 - 0.05 = 0.95$
- $P(X \leq x) = P(Z \leq z) = 0.95 \Longrightarrow z = \frac{1.64+1.65}{2} = 1.645$
- $z = \frac{x-\mu}{\sigma} \Longrightarrow x = \mu + \sigma z = 10 + 2 \times 1.645 = 13.29$
- The supervisor nurse choose approximately 13 minutes for the cut off.

[Question 2]

The GMAT scores of all examinees who took that test this year produce a distribution that is approximately normal with a mean of 420 and a standard deviation of 32.

a) Find the probability that the score of a randomly selected examinee is within 2.5 of the population standard deviation.
- X: The GMAT score.
- $X \sim N(420, 32)$.
- $P(\sigma - 2.5 < X < \sigma + 2.5) = P(29.5 < X < 34.5) = P(-12.20 < Z < -12.05) =$ $P(Z < -12.20) - P(Z < -12.05) = 0.000 - 0.000 = 0$.

b) Find the probability that the score of a randomly selected examinee is more than 400.

- $P(X > 400) = P\left(z > \frac{400-420}{32}\right) = P(z > -0.625) = P(z > -0.62) = 1 - P(Z < -0.62) = 1 - 0.2676 = 0.7324.$

c) What is the minimum score that an examinee must get so that his/her score belongs to the top 10% scores.

- $P(X < P_{90}) = 0.9.$
- $P(Z < z) = 0.9 \Longrightarrow z = 1.28.$
- $P_{90} = \mu + z\sigma = 420 + 1.28 \times 32 = 460.96.$

d) What are the mode and the median of the distribution?

- Mode=Median= $\mu = 420.$

[Question 3]

The amount of time taken by a bank teller to serve a randomly selected customer has a normal distribution with a mean of 2 minutes and a standard deviation of 0.5 minutes.

a) Find the probability that a randomly selected customer will take more than 2.5 minutes to be served.

- X: The amount of time taken by a bank teller to serve a customer.
- $X \sim N(2, 0.5).$
- $P(X > 2.5) =?$
- $z = \frac{x-\mu}{\sigma} = \frac{2.5-2}{0.5} = 1.$
- $P(X > 2.5) = P(Z > 1) = 1 - P(Z < 1) = 1 - 0.8413 = 0.1587.$

b) Find the probability that a randomly selected customer will take more than 1 minute and less than 2.5 minutes to be served.

- $P(1 < X < 2.5) =?$
- $z = \frac{x-\mu}{\sigma} = \frac{1-2}{0.5} = -2$ and $z = \frac{2.5-2}{0.5} = 1.$
- $P(1 < X < 2.5) = P(-2 < X < 1) = P(Z < 1) - P(Z < -2) = 0.8413 - 0.0228 = 0.8185.$

c) Find the probability that a randomly selected customer will be served within 1.5 standard deviation of the population mean.

- $P(2 - 1.5 \times 0.5 < X < 2 + 1.5 \times 0.5) = P(1.25 < X < 2.75) =?$
- $z = \frac{x-\mu}{\sigma} = \frac{1.25-2}{0.5} = -1.5$ and $z = \frac{2.75-2}{0.5} = 1.5.$
- $P(1.25 < X < 2.75) = P(-1.5 < Z < 1.5) = P(Z < 1.5) - P(Z < -1.5) = 0.9332 - 0.0668 = 0.8664.$
- Or $P(-1.5 < Z < 1.5) = 1 - 2P(Z < -1.5) = 1 - 2 \times 0.0668 = 0.8664.$

d) Find the probability that a randomly selected customer will be served within 3 minutes of the population mean.

- $P(2 - 3 < X < 2 + 3) = P(-1 < X < 5) =?$

- $z = \frac{x-\mu}{\sigma} = \frac{-1-2}{0.5} = -6$ and $z = \frac{5-2}{0.5} = 6$.
- $P(-1 < X < 5) = P(-6 < Z < 6) \approx 1$.
- Or $P(-6 < Z < 6) = P(Z < 6) - P(Z < -6) \approx 1 - 0 = 1$.

e) What percent of customers are served in less than 3 minutes?
- $P(X < 3) =?$
- $z = \frac{x-\mu}{\sigma} = \frac{3-2}{0.5} = 2$.
- $P(X < 3) = P(Z < 2) = 0.9772$.
- 97.72% of customers are served in less than 3 minutes.

f) Find the median of this distribution.
- The normal distribution is symmetric. The median is the same as the mean. Therefore, the median is 2 minutes.

g) Find the first quartile of this distribution.
- We need to find x so that $P(X < x) = 0.25$ $(x = P_{25} = Q_1)$.
- We first find z so that $P(Z < z) = 0.25$.
- Using the table $z \approx -0.67$ (0.25 is not in the table. The closet value to 0.25 is 0.2514 that corresponds to $z = -0.67$).
- Therefore $Q_1 = P_{25} = x = \mu + z\sigma = 2 + (-0.67) \times 0.5 = 2 - 0.335 = 1.665$ minutes.

h) Find the 80th percentile of the probability distribution.
- We need to find x so that $P(X < x) = 0.8$ $(x = P_{80})$.
- We first find z so that $P(Z < z) = 0.8$.
- Using the table $z \approx 0.84$ (0.8 is not in the table. The closet value to 0.8 is 0.7995 that corresponds to $z = 0.84$).
- Therefore, $P_{80} = x = \mu + z\sigma = 2 + 0.84 \times 0.5 = 2.42$ minutes.

i) Find the probability that both of two randomly selected customers will take less than 3 minutes each to be served.
- X_1: The amount of time taken by a bank teller to serve customer #1.
- X_2: The amount of time taken by a bank teller to serve customer #2.
- X_1 and X_2 are independent.
- We use also the result of part e).
- $P(X_1 < 3 \text{ and } X_2 < 3) = P(X_1 < 3) \times P(X_2 < 3) = 0.9772 \times 0.9772 = 0.9549$.

[Question 4]

The management of a supermarket wants to adopt a new promotional policy of giving a free gift to every customer who spends more than a certain amount per visit at this supermarket. The expectation of the management is that after this promotional policy is advertised, the expenditures for all customers at this supermarket will be normally distributed with a mean of \$115 and a standard deviation of \$26. If the management wants to give free gifts to at most 13.35% of the customers, what should the amount be above which a customer would receive a free gift?

- X: The expenditure for a randomly selected customer at this supermarket.
- $X \sim N(115, 26)$.
- We need to find x so that $P(X > x) = 0.1335$ or $P(X < x) = 1 - 0.1335 = 0.8665$.
- We first find z so that $P(Z < z) = 0.8665$.
- Using the table $z = 1.11$.
- Therefore $x = \mu + z\sigma = 115 + 1.11 \times 26 = 115 + 28.86 = 143.86$.
- A customer would receive a free gift for an amount above $143.86.